Mathematics Review AP Chemistry

Although the mathematics on the AP chemistry exam is not difficult, students find it to be challenging because most of it requires basic arithmetic skills that you have not used since middle school (or even elementary school). Students generally have a good sense of how to use a calculator, but lack the skills of doing math without a calculator. Most math classes stress the use of calculators to solve problems and on the AP chemistry exam at least **fifty percent** of it is to be done **without** calculators. You will need to practice the basic math skills that you already know (just haven't used much in the past) to develop your thinking to solve the problems that you will encounter.

Format of Multiple Choice AP test:

- 60 multiple choice questions
- 90 minutes on the section
- Average time is 90 seconds per question
- **NO CALCULATORS ARE TO BE USED**
- **NO GUESSING PENALTY**
- Choices go from the
  - lowest number to the highest number
  - they are generally lined up by decimal points, even though this does not make the number list "straight"
    - Allows you to see decimal points and significant figures easier.
- Periodic table and formula charts are provided at the beginning of the exam
- Questions will often be grouped by common given information, but every question will have it's own four answer choices

Source of these problems

Each problem of the following practice problems are directly related to released AP chemistry exam questions. There will be nothing in this packet that does not directly relate to developing the skills and abilities that students need to be successful on the AP chemistry exam.

How to do the math

What follows are the notes and the "meat" of this packet. The intent of this packet is to cover the math necessary to be successful on the AP Chemistry exam. If it seems to be overly basic in its approach, the underlining principle in writing this packet is to assume that you (the students) do not remember anything, so we will start at beginning.

Conversion Factors and Dimensional Analysis

There are two guiding mathematical principles when working with conversion factors.

1. When the numerator (the top of a fraction) and the denominator (bottom of the fraction) are the same, then the fraction equals one.
2. When any number is multiplied by one, you do not change the number at all.

A conversion factor is a fraction that equals one, since the top and the bottom are the same thing, just expressed in different units. Examples of conversion factors are:

$$\frac{1\text{ dollar}}{10\text{ dimes}},\ \frac{12\text{ inches}}{1\text{ foot}},\ \frac{365\text{ days}}{1\text{ year}},\ \frac{5280\text{ feet}}{1\text{ mile}},\ \frac{12\text{ eggs}}{1\text{ dozen}},\ \frac{1760\text{ yards}}{5280\text{ feet}},\ \frac{5280\text{ feet}}{1760\text{ yards}}.$$

For the last two, how do you know which one to use? You let the units guide you. You ask yourself a series of questions as you do the problem.

1. What unit has to go on the bottom to cancel?
2. What can I change that unit into?
3. What numbers will make them equal?

Using the units to guide you in the problem is called "dimensional analysis". This method only works if you put your units in the problem and cancel them. Here is the trick…you have to think about them canceling. Don't just make your teacher happy by canceling units. If you do not get the units to work out and give you what you are asked to find, then you have 100% of getting the problem wrong. If the units do work out, your final unit is what you are asked to find, then you have a 90% chance of getting the problem right (the other 10% is making dumb mistakes or math errors in your calculations).

Say you are 16 years old and you want to know how old you are in minutes. So you start out with 16 years.

$$16\ \cancel{\text{years}} \times \frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}} \times \frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}} \times \frac{60\text{ minutes}}{1\ \cancel{\text{hour}}} = 8\,409\,600\text{ minutes}$$

1. What unit has to go on the bottom to cancel? Answer: years
2. What can I change that unit into? Answer: days (you could have picked months or weeks)
3. What numbers will make them equal? Answer: 1 year is 365 days, so 1 is on the bottom and 365 is on the top.

Notice that the top and the bottom of the fraction measure the same thing, so this conversion factor equals one.

1. What unit has to go on the bottom to cancel? Answer: days
2. What can I change that unit into? Answer: hours
3. What numbers will make them equal? Answer: 1 day is 24 hours, so 1 is on the bottom and 24 is on the top.

Notice that the top and the bottom of the fraction measure the same thing, so this conversion factor equals one.

1. What unit has to go on the bottom to cancel? Answer: hours
2. What can I change that unit into? Answer: minutes
3. What numbers will make them equal? Answer: 1 day is 60 minutes, so 1 is on the bottom and 60 is on the top.

Notice that the top and the bottom of the fraction measure the same

You want to make sure that you cancel as you go. If the units don't cancel, you made a mistake. Notice also that all that this problem did was to multiple 16 years by 1, 1 and 1. Anytime that you multiple by one, you do not change the number.

A common mistake is that students put the number from the start at the bottom of the first conversion factor, making the problem look like this $16\ \text{years} \times \frac{5840\,\text{days}}{16\,\text{year}}$. You should never have to reach for a calculator to put the numbers in a conversion factor – you should know what the numbers are and where the numbers come from!

You were asked for your age in minutes but were given years, so you have to convert the units. If the final unit is minutes then you have a 90% chance that you got the problem correct.

Another method to set up the problem involves "railroad tracks" which is shown below. This method is the same thing; it is just set up differently. Either setup will get you credit on the AP exam and earn you the point for showing your work.

16 ~~years~~	365 ~~days~~	24 ~~hours~~	60 minutes	= 8 409 600 minutes
	1 ~~year~~	1 ~~day~~	1 ~~hour~~	

Note on using calculators

Solve this problem: $3.85 \times \frac{9.11 \times 10^{-31}}{1.60 \times 10^{-19}} \times \frac{3}{25} \times \frac{6.022 \times 10^{23}}{35.4527} \times \frac{55.85}{96500} =$ _____

Students commit two common errors when they reach for their calculator to solve problems like the ones above. First, the calculator cannot read your mind and will do exactly what you tell it to. If students are not careful, order of operations will cause you grief. Many students set up these problems correctly on the free response and then do not earn points because they cannot use their calculator correctly.

The second problem is entering scientific notation into your calculators. Take the most common number in scientific notation used in chemistry, 6.022×10^{23}. The "6.022" can be called the *coefficient*, *digit term* or the *significand* while the "x 10^{23}" is called the *base* or the *exponential term*. If you use the buttons "10^" or "^" to enter scientific notation, you SHOULD (and in some problems MUST) use parenthesis around the numbers to "glue" the coefficient to the base, if you do not, you may get the wrong answer (depends on the type of problem you are solving). A much better method to enter this information into your calculator is to use the "EE" key (graphing calculators), "EXP" key (generally on non-graphing calculators, but some may have the "EE" key), "E" key (not generally seen) and the "x10" (not generally seen). By using this key, the calculator automatically "glues" the coefficient with the base.

When doing this type of problem, there are two things you want to do…use the EE key and use parenthesis around the top numbers (3.85, 9.11×10^{-31}, 3, 6.022×10^{23} and 55.85), hit the divide key, and then use parenthesis around the bottom numbers (1.60×10^{-19}, 25, 35.4527 and 96500). This will save you many keystrokes on the calculator. Reducing the number of keystrokes reduces the number of chances that you have to make a mistake.

```
3.85*9.11*10^-31
/1.60*10^-19*3/2
5*6.022*10^23/35
.4527*55.85/9650
0
  2.58599493E-31
```

Version One

```
(3.85*9.11E-31*3
*6.022E23*55.85)
/(1.60E-19*25*35
.4527*96500)
     25859949.32
```

Version Two

```
3.85*9.11*10^-31
*3*6.022*10^23*5
5.85
  3.538867398E-4
Ans/(1.60*10^-19
*25*35.4527*9650
0)
     25859949.32
```

Version Three

```
3.85*9.11*10^-31
*3*6.022*10^23*5
5.85
  3.538867398E-4
1.60*10^-19*25*3
5.452*96500
   1.3684472E-11
3.538867398E-4/1
.3684472E-11
     25860459.93
```

Version Four

DID YOU KNOW

That a horizontal line in mathematics is called a **vinculum?** Its name comes from the Latin and it means "bond" or "tied". It is used in a mathematical expression to indicate that the group is to be considered "tied" together. The vinculum can be used to express division. The numerator appears above the vinculum and the denominator beneath it.

Because the vinculum means to group together, we have to use the parenthesis and the "EE" key to glue the correct parts together so the calculator can do the problem correctly.

Version one is wrong. The student did not use the parenthesis to "glue" the coefficient to the base.

Version two is correct. Version three and four are also correct, however, version two only required 60 keystrokes, while version three required 70 and version four 91. The fewer keystrokes, the less likely it is that you will make a mistake. The slight difference in the answers for version two/three and version four is due to the fact that the numbers in two and three were carried in the memory of the calculator and not entered by the student.

Note on Canceling

As problems are done in this packet, when numbers or units are canceled, not only will the units cancel, but how the units are canceled, will cancel. The ways to show units canceling are "/", "\", "—" and "|". The "/" was done with the years above, "\" canceled the days and the "—" canceled the hours. This is done so that it is easier for students to follow what was canceled in the example problems.

Cross Canceling of Numbers

Cross canceling refers to canceling a numerator with a denominator, or a factor that is in each of them. It is one of the most important skills you need to remember for dealing with the multiple choice problems on the AP chemistry exam. In the first example, the numbers that cancel are side by side, however, on the exam, the numbers may not be side by side. Just remember to cancel numerator with denominator.

Example: $\frac{12}{5} \times \frac{5}{3} \rightarrow \frac{12}{\cancel{5}_{1}} \times \frac{\cancel{5}^{1}}{3} \rightarrow \frac{\cancel{12}^{4}}{\cancel{5}_{1}} \times \frac{\cancel{5}^{1}}{\cancel{3}_{1}} = 4$

the fives cancel

3 goes into 3, once and the 3 goes into 12 four times.

Example: $7 \times \frac{3}{2} \times \frac{4}{8} \times \frac{3}{9} \times \frac{4}{14} \times \frac{1}{3} = \rightarrow \frac{\cancel{7}^{1}}{1} \times \frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{3}^{1}}{\cancel{9}_{3}} \times \frac{\cancel{4}^{1}}{\cancel{14}_{2}} \times \frac{1}{\cancel{3}_{1}} = \frac{1}{3 \times 2} = \frac{1}{6}$

Memorization

Although this is AP chemistry, you must remember some basic math and arithmetic information. What you have to memorize (and apply) is kept to a minimum; but you **MUST** know it to be successful on the AP chemistry exam.

The first thing to remember is:

- Divided by ten, decimal point moves to the left one place.
- Multiple by ten, decimal point moves to the right one place.
- Divide by powers of ten ($100 = 10^2$, $1000 = 10^3$, etc) move the decimal point to the left the same number of spaces as the power of ten.
- Multiple by powers of ten ($100 = 10^2$, $1000 = 10^3$, etc) move the decimal point to the right the same number of spaces as the power of ten.

Another common mathematic problem is division by a fraction. Remember, find the main division bar and rewrite as a multiplication problem by multiplying by the reciprocal of the fraction on the denominator:

$$\frac{\frac{a}{b}}{c} = a \times \frac{c}{b} = \frac{ac}{b}$$

To make problems easier, it is generally better to write a mixed fraction as an improper fraction. Remember $a\frac{b}{c} = \frac{(ac)+b}{c}$ example is $3\frac{5}{8} = \frac{(3 \text{ x } 8)+5}{8} = \frac{29}{8}$

A common approach for a problem might be to have you solve the following problem, $\frac{3.00}{1.20}$ which can be done "long hand" or you can use the fraction information and division by a fraction to make the problem a little easier.

$$\frac{3.00}{1.20} = \frac{3.00}{1\frac{1}{5}} = \frac{3.00}{\frac{6}{5}} = 3.00 \times \frac{5}{6} = \frac{3.00}{6} \times 5 = \frac{1}{2} \times 5 = 2.50$$

The following fractions, their decimal equivalence and their percentages must be memorized.

$\frac{1}{5} = 0.20 = 20.0\%$	$\frac{2}{5} = 0.40 = 40.0\%$	$\frac{3}{5} = 0.60 = 60.0\%$	$\frac{4}{5} = 0.80 = 80.0\%$
$\frac{1}{8} = 0.125 = 12.5\%$	$\frac{3}{8} = 0.375 = 37.5\%$	$\frac{5}{8} = 0.625 = 62.5\%$	$\frac{7}{8} = 0.875 = 87.5\%$
$\frac{1}{3} = 0.33 = 33\%$	$\frac{2}{3} = 0.67 = 67\%$	$\frac{1}{4} = 0.25 = 25\%$	$\frac{3}{4} = 0.75 = 75\%$
$\frac{1}{2} = 0.50 = 50\%$			

Once you know these fractions, you can use them to determine other fractions.

Example: What is $\frac{1}{6}$ as a decimal?

Think of $\frac{1}{6}$ as $\frac{1}{2}$ of $\frac{1}{3}$ → ½ of 0.333 = 0.1667

Likewise, you can think of $\frac{1}{8}$ as ½ of ¼, which gives you ½ x ¼ or ½ x 0.25 = 0.125. You can think of $\frac{5}{8}$ as $\frac{4}{8} + \frac{1}{8} = \frac{1}{2} + \frac{1}{8}$ = 0.5 + 0.125 = 0.625, so as long as you have the basic list memorized, you should be able to do problems that appear on the test.

Example: What is 0.025 as a fraction?

First thing is to recognize that the fraction is really based on 0.25, or ¼. But you want 0.025, so that is done like this: $0.025 = \frac{0.25}{10} = \frac{1/4}{10} = \frac{1/4}{10/1} = \frac{1}{4} \text{ x } \frac{1}{10} = \frac{1}{40}$

This type of problem often appears when the question is dealing with stoichiometry or titration problems where you are given molarities like 0.025 M, 0.0125 M and 0.020 M.

If you are given a decimal number like 0.150 M or 0.120 M and it does not fit any of the fractions above, you can also write it as a fraction by moving the decimal to the right until it is behind the last non-zero number, and then put it over the appropriate power of 10, which would be $10^{(\text{number of decimal places moved})}$. So if you move a decimal 3 places to the right, then it the denominator will become 10^3, or 1000.

So, 0.150 M would become $\frac{15}{100}$ and 0.120 M becomes $\frac{12}{100}$. If you have a problem that involves molarities like these, work the problem with the fractions; the problem is designed for you to do that.

Example is from the 1994 AP test question #55.

What volume of 0.150-molar HCl is required to neutralize 25.0 millilters of 0.120-molar $Ba(OH)_2$?

(A) 20.0 mL
(B) 30 0 mL
(C) 40.0 mL
(D) 60.0 mL
(E) 80.0 mL

$$\text{Molarity} = \frac{moles_{solute}}{liters_{solution}}$$

First step is to write out the balance equation.

$$Ba(OH)_2 + 2\ HCl \rightarrow 2\ HOH + BaCl_2$$

The second step is to place your information so you can do the problem.

$$Ba(OH)_2 + 2\ HCl \rightarrow 2\ HOH + BaCl_2$$

25.0 mL	? mL
0.120 M	0.150 M

Since the problem said "neutralize" we know that the moles of acid and the moles of base are stoichiometrically equal.

So, the first thing to do is to rewrite the molarities as fractions, $\frac{15}{100}$ and $\frac{12}{100}$.

Next, find the moles of $Ba(OH)_2$, moles = molarity x volume.

$$\text{moles} = \text{molarity} \times \text{volume} = \left(\frac{12}{100}\right)\frac{\text{moles}}{\cancel{\text{liter}}} \times 25.0\ \text{milli}\cancel{\text{liters}} = \left(\frac{12}{\cancel{100}_{4}}\right)\left(\frac{\cancel{25}}{1}\right) = \frac{12}{4} = 3\ \text{millimoles}$$

Next find the number of moles HCl.

$$3.0\ \text{milli}\cancel{\text{moles Ba(OH)}_2} \times \frac{2\ \text{moles HCl}}{1\ \cancel{\text{mole Ba(OH)}_2}} = 6\ \text{millimoles HCl}$$

Last step is to find the volume of HCl used. Volume equals $\frac{moles}{molarity}$.

Notice that I only cancelled the "liters" part of the milliliters. This would leave a "milli" and that will be multiplied by the moles to get millimoles. You can "track" the "milli" through

$$\text{volume} = \frac{\text{moles}}{\text{molarity}} = \frac{6\ \text{millimoles HCl}}{\left(\frac{15}{100}\right)\left(\frac{\text{moles}}{\text{liters}}\right)} = \cancel{6}^{2}\ \text{milli}\cancel{\text{moles}}\ \text{HCl} \times \left(\frac{\cancel{100}^{20}}{\cancel{15}_{\cancel{5}\ 1}}\right)\left(\frac{\text{liters}}{\cancel{\text{moles}}}\right) = 40.0\ \text{milliliters}$$

A common type of problem will require you to use scientific notation and to use the following exponent laws (which need to be memorized):

$a^m \times a^n = a^{m+n}$ $\quad a^m \div a^n = a^{m-n}$ $\quad \left(a^m\right)^n = a^{m \times n}$ $\quad (ab)^m = a^m b^m$

$a^0 = 1$ $\quad a^{-n} = \frac{1}{a^n}(a \neq 0)$

Example: A problem gives you the Keq value of 2.0 x 10^7 and would like the Keq value for the reverse reaction, which is the reciprocal of the given Keq value.

Answer: $$\frac{1}{2.0\times10^7} = \frac{1\times10^0}{2.0\times10^7} = \frac{1}{2}\times10^{0-7} = 0.5\times10^{-7} = 5\times10^{-8}$$

Another way to do this problem is to rewrite 1 as 10 x 10^{-1}. Doing this, you will have:

$$\frac{1}{2.0\times10^7} = \frac{10\times10^{-1}}{2.0\times10^7} = \frac{10}{2.0}\times10^{-1-7} = 5\times10^{-8}$$

This trick is very helpful when you have to do division. It allows you to get a whole number directly from the problem and not have to try to move the decimal point in scientific notation (like the first method does).

The last exponent rule is commonly used on the AP exam with units (both on the formula charts and in problems). The exponent law that says $a^{-n} = \frac{1}{a^n}$ will often show up on the test with units like sec^{-1} (which means $\text{sec}^{-1} = \frac{1}{\text{sec}^1} = \frac{1}{\text{sec}}$) or mol^{-1} ($\frac{1}{\text{mol}}$) or K^{-1} ($\frac{1}{\text{K}}$). When you are doing kinetic problems, you will see units like, M^{-2} ($M^{-2} = \frac{1}{M^2} = \frac{1}{\frac{\text{mol}^2}{\text{liter}^2}} = 1 \text{ x } \frac{\text{liters}^2}{\text{mol}^2} = \frac{\text{liters}^2}{\text{mol}^2}$).

You will also need to remember your "perfect squares" from 1 to 12.

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	$6^2 = 36$
$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$	$11^2 = 121$	$12^2 = 144$

Scientific Notation

In science, the numbers are characteristically very large (a mole is 602 214 179 000 000 000 000 000) or very small (the charge carried by an electron is 0.000 000 000 000 000 000 160 217 653 coulombs). Scientific notation is used to conveniently write the numbers using powers of ten. So 1,650,000 can be written (with four significant figures) as 1.650 x 10^6 (which generally will be written as 1.650 x 10^6 ~ it makes it easier for people to see the exponent). This can be thought of as 1.650 x 10 x 10 x 10 x 10 x 10 x 10 (six sets of 10, since the exponent was 6).

Notice that

$$
\begin{aligned}
&1.650 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\
&= 16.50 \times 10 \times 10 \times 10 \times 10 \times 10 \\
&= 165.0 \times 10 \times 10 \times 10 \times 10 \\
&= 1650. \times 10 \times 10 \times 10 \\
&= 16500. \times 10 \times 10 \\
&= 165000. \times 10 \\
&= 1{,}650{,}000
\end{aligned}
$$

A note about commas and numbers: Most textbooks will not use commas to separate numbers into groups of three. They will add a space instead, so 1,650,000 will be written as 1 650 000 – this is because in other countries, a comma in a number is read as a decimal point and you can't have two decimal points in a number!

So by multiplying by 10, we are moving the decimal place over to the right that many times and get a number bigger than one.

1.650×10^6 is equal to 1. 6 5 0 0 0 0

Likewise, if we have a number that is smaller than one, we should divide by powers of 10 and move the decimal point to the left. So 0.000321 can be thought of as:

$$\frac{3.21}{10 \times 10 \times 10 \times 10} = \frac{3.21}{10^4} = \frac{3.21 \times 10^0}{10^4} = 3.21 \times 10^{0-4} = 3.21 \times 10^{-4}$$

Remember that 10^0 is 1 and anything multiplied by one is still itself.

Students have a problem sometimes in remembering if the exponent is positive or negative. The easiest way to remember this is to ask yourself "is the original number less than one?" The answer to this question will determine if your exponent is positive or negative.

Is the original number less than one?	Exponent will be	Examples
Yes	Negative	$0.0025 = 2.5 \times 10^{-3}$
No	Positive (or zero)	$320500 = 3.205 \times 10^5$

Powers and Roots

Just as operations of addition and subtraction and the operations of multiplication and division are related to each other (in math terms they are inverse operations of each other), so are exponents and roots. When $\sqrt[2]{9}$ is written, the 2 is called the "index", the 9 is called the "base" or "radicand" and the $\sqrt{\ }$ is called the "radical" or "root". When the index is "2", we often call it "square root".

Fractional exponents can be used to represent taking the "root" of a number, thus $\sqrt[2]{9}$ can also be written as $9^{1/2}$, likewise $8^{1/3}$ is another way to write the third root of eight, $\sqrt[3]{8}$, which equals 2. We will use this idea to solve problems like $x^2 = 2.5 \times 10^{-9}$. The idea here is to write the scientific notation in a non-traditional form so that it will be easier to take the square root.

$x^2 = 2.5 \times 10^{-9} \rightarrow x^2 = 25 \times 10^{-10} \rightarrow x = \sqrt{25 \times 10^{-10}} \rightarrow x = \left(25 \times 10^{-10}\right)^{1/2} \rightarrow x = (25)^{1/2} \times \left(10^{-10}\right)^{1/2}$

Since we went from 2.5 to 25 (we want a whole number that is a perfect square), we moved the decimal point to the right; we will need to move it back one extra space to the left, so the exponent will have to become -10.

$25^{1/2}$ is the square root of 25, which is 5 and the $\left(10^{-10}\right)^{1/2} = 10^{-5}$ since a power to a power is multiplied and -10 x ½ = -5. So the final answer is $x = 5 \times 10^{-5}$.

Logarithms

Another type of math problem that you will be expected to be able to do is simple logarithms. Remember a logarithm is just another way to write an exponent problem. Remember that logarithms are just a "circular" way of writing exponents. Take for example $2^x = 8$ can be written as $\log_2 8$.

$$2^x = 8 \rightarrow \log_2 8 = 3$$

For the pH = 3.45, the "3" is called the characteristic and the ".45" is the mantissa. The characteristic is **ALWAYS** determined by the exponent in the number. The mantissa is always determined by the number in front of "x" in scientific notation (hence the special rule for significant figures).

Say you have $\log_{10} 5.16 \times 10^{-9}$; the -9 will determine the characteristic and the 5.16 will provide you the mantissa.

Remember that $-\log_{10} 1.0 \times 10^{-B} = B$, so when given a question that says "determine the approximate range that an indictor would be appropriate for?" – you are looking for the pKa value; you will need to know how to find the approximate answer. The exponent will always give you the characteristic of the number…unless the number in front of the "x" sign is 1, then your answer (when you take the log) will be (exponent – 1) – in this case B - 1.

Look at the following chart and look at the pattern of the numbers:

Number	**$\log_{10}$ of number**
1.00×10^{-5}	5.00
1.78×10^{-5}	4.75
3.16×10^{-5}	4.50
5.62×10^{-5}	4.25
10.0×10^{-5}	4.00

The logarithmic scale is not linear. The half-way point for the log (a mantissa of ".5") will be determined by $3.16 \times 10^{-??}$; an easy way to remember this is $\pi \times 10^{-??}$ will give you the mantissa of ".5" and the exponent determines the characteristic of the your answer.

When you see things like pH, pOH, pK_a, pK_b or pK_w, what does the little "p" represent? The little "p" in front of a term means "$-\log_{10}$", so $pH = -\log_{10}[H^+]$, likewise $pK_a = \log_{10}K_a$ and so on.

Manipulation of Equations

In chemistry, just as in other science classes, students will have to manipulate equations. The equations are not hard and there are not too many that appear on the test. The most common equations and where they are used are given below.

Equation	Where is it used
$D = \frac{m}{V}$	Density problems – usually multiple choice
PV=nRT	Ideal Gas Law – gas law problems, usually on the free response
$M = \frac{\text{moles}_{\text{solute}}}{\text{liters}_{\text{solution}}}$	Molarity – determine the concentrations of solutions, usually multiple choice
$q = mc\Delta T$	Thermochemistry/specific heat problem – both multiple choice and free response problems
$\Delta G = \Delta H - T\Delta S$	Thermodynamics problems – both multiple choice and free response problems
$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$	Combined Gas Law – gas law problems, multiple choice usually

When solving an equation…use the algebra that you already know. You want to solve for a term, (meaning you want that term by itself) your equation should look like "term = variables".

Remember to solve for a variable, look at the other variables and you want to use the operation that "undoes" whatever you have in the problem.

- division undoes multiplication
- multiplication undoes division
- addition undoes subtraction
- subtraction undoes addition
- power undoes a logarithm
- logarithm undoes a power

Examples:

Solve for "m"

$$D = \frac{m}{V}$$
$$VD = \frac{m}{V} \times V$$
$$VD = m$$

Solve for "m"

$$q = mc\,\Delta T$$
$$\frac{q}{c\,\Delta T} = \frac{mc\,\Delta T}{c\,\Delta T}$$
$$\frac{q}{c\,\Delta T} = m$$

Solve for "volume"

$$M = \frac{\text{moles}_{\text{solute}}}{\text{liters}_{\text{solution}}} \Rightarrow$$
$$M = \frac{\text{mol}}{\text{vol}}$$
$$M \times \text{vol} = \frac{\text{mol}}{\text{vol}} \times \text{vol}$$
$$M \times \text{vol} = \text{mol}$$
$$\frac{M \times \text{vol}}{M} = \frac{\text{mol}}{M}$$
$$\text{vol} = \frac{\text{mol}}{M}$$

Solve for "T_2"

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$\frac{T_2P_1V_1}{T_1} = \frac{T_2P_2V_2}{T_2}$$
$$\frac{T_2P_1V_1}{T_1} = \frac{P_2V_2}{}$$
$$\left(\frac{T_1}{P_1V_1}\right)\left(\frac{T_2P_1V_1}{T_1}\right) = \left(\frac{P_2V_2}{}\right)\left(\frac{T_1}{P_1V_1}\right)$$
$$T_2 = \frac{P_2V_2T_1}{P_1V_1}$$

The last two are the hardest for students to do, since the problem asked for you to solve for a variable in the denominator. Your first task is to get the term out of the denominator, i.e. multiple both sides by that term and have them cancel. Then use algebra to solve for that term.

When you have the equation solved and are substituting (plugging in) your values, please don't forget the units. The units will tell you if you have the equation correct. If the units don't work out, you know that you made a mistake in solving the equation. Say that you are using the density equation to solve for volume and did the following work:

$$D = \frac{m}{V}$$

$$\left(\frac{1}{m}\right)(D) = \left(\frac{m}{V}\right)\left(\frac{1}{m}\right)$$

$$\frac{D}{m} = V$$

and then substituted the numbers with units and got this:

$$\frac{D}{m} = V \rightarrow \frac{\frac{\text{grams}}{\text{milliliters}}}{\text{grams}} = \frac{\frac{\text{grams}}{\text{milliliters}}}{\frac{\text{grams}}{1}} = \left(\frac{\text{grams}}{\text{milliliters}}\right)\left(\frac{1}{\text{grams}}\right) = \frac{1}{\text{milliliters}}$$

You are looking for volume and should get the units of "milliliters" and when you solved the problem, you got the units "$\frac{1}{\text{milliliters}}$" which should tell you that your equation is wrong! Not only that, but getting the units "$\frac{1}{\text{milliliters}}$" tells you how to fix your equation. You want "milliliters" and you got the reciprocal of that unit, so all you need to do is to take the reciprocal of your equation (flip the equation) and you will have $V = \frac{m}{D}$.

Some math tricks

- If you don't like to work with numbers like 0.025, move the decimal point three places to the right and think of the number as 25 – just remember to move the decimal point three places back to the left when done with the problem.
- Anytime that you have a number that has 25 in it…think of it as money. You are dealing with a quarter. If the problem is 0.25x = 1.25 → that is the same as asking how many quarters does it take to make $1.25?
- If you are a "music person" – then think of "25" as a quarter note, so 0.25x = 1.25 is the same as asking, "How many quarter notes does it take to make a whole and quarter note?" – make it relevant to your interests.
- If you are squaring a number that ends in 5, like 35. This is what you can do.
    - Take the number in front of the 5 – in this case it is 3.
    - Add one to the number – in this example that would be 1 + 3 = 4
    - Take this number and multiple by the original number; here we get 4 x 3 = 12.
    - Take this number and put 25 at the end. Here we would get 1225.

- Use the skills that you learned in algebra for factoring.
  - $(a + b)(a - b) = a^2 - b^2$ is one of the most common factoring methods used in algebra. How does it relate to doing problems? Let's say that you have the following problem: 47 x 43 = ? You can think of that as:

$$(45 + 2)(45 - 2)$$
$$= 45^2 - 2^2$$
$$= 2025 - 4$$
$$= 2021$$

  - Another method is to use the distributive property "backwards". Let's say that you have to solve the problem 12 x 14. To make this problem simpler to solve, think of 14 as 12 + 2, so:

$$12 \times 14 = ?$$
$$12(12 + 2) = ?$$
$$(12 \times 12) + (12 \times 2) = ?$$
$$12^2 + (12 \times 2) = ?$$
$$144 + 24 = 168$$

Problems

DO NOT use a calculator on these problems. This should all be able to be done without them.

1) Complete the following chart – have the fractions in lowest terms.

	Decimal	Fraction		Decimal	Fraction
a)	0.375		j)	0.67	
b)	0.75		k)	0.125	
c)	0.875		l)	0.33	
d)	0.60		m)	0.5	
e)	0.25		n)	0.20	
f)	0.020		o)		$\frac{3}{4}$
g)	0.075		p)		$\frac{1}{4}$
h)	0.005		q)		$\frac{1}{5}$
i)	0.625		r)		$\frac{1}{16}$

2) Solve the following by rewriting them as fractions (if needed) and **show your work**.

Express answers in this column as a fraction or whole number		Express answers in this column as a decimal (may approximate if needed)	
a)	$\frac{0.5}{0.125}$	g)	$\frac{1}{1.25}$
b)	$\frac{0.25}{0.50}$	h)	$\frac{0.5}{0.2}$
c)	$\frac{0.025}{0.075}$	i)	$\frac{1/8}{1/5}$
d)	$\frac{0.125}{0.075}$	j)	$\frac{1}{2\frac{1}{5}}$
e)	$\frac{0.6}{0.02}$	k)	$\frac{3/8}{2.5}$
f)	$\frac{0.6}{0.2}$	l)	$\frac{2.625}{1.75}$

3) Solve the following, **<u>showing all of your work.</u>**

a)	$\frac{6 \text{ x } 10^{18}}{4 \text{ x } 10^{-5}} =$
b)	$\frac{1}{4 \text{ x } 10^{-5}} =$
c)	$\frac{(4 \text{ x } 10^{-5})(1.5 \text{ x } 10^{13})}{1.5 \text{ x } 10^{4}} =$
d)	$(4 \text{ x } 10^{-5})(1.5 \text{ x } 10^{13}) =$
e)	$\frac{(2 \text{ x } 10^{7})(1.5 \text{ x } 10^{4})}{4.5 \text{ x } 10^{8}} =$
f)	$(4 \text{ x } 10^{-5})^{3} =$

4) Solve the following problems, using cross canceling of numbers. **<u>Show your work.</u>**

a) $9 \times \frac{1}{18} \times \frac{2}{4} \times \frac{44}{1} =$ _____

b) $280 \times \frac{1}{28} \times \frac{3}{1} \times \frac{6}{1} =$ _____

c) $70 \times \frac{1}{28} \times \frac{1}{1} \times \frac{42}{1} =$ _____

d) $48 \times \frac{1}{32} \times \frac{2}{3} \times \frac{158}{1} =$ _____

e) $87 \times \frac{1}{174} \times \frac{3}{2} \times \frac{28}{1} =$ _____

f) $12 \times \frac{1}{2} \times \frac{1}{2} \times \frac{42}{1} =$ _____

g) $165 \times \frac{1}{55} \times \frac{2}{4} \times \frac{158}{1} =$ _____

h) $0.33 \times \frac{1}{44} \times \frac{1}{1} \times \frac{100}{1} =$ _____

5) Solve for "*x*".

a) $\frac{(x)(x)}{0.5} = 5.0 \times 10^{-5}$

b) $\frac{(x)(x)}{0.25} = 6.4 \times 10^{-7}$

c) $\frac{(x)(x)}{0.125} = 3.2 \times 10^{-9}$

d) $(x)(2x)^2 = 3.2 \times 10^{-8}$

e) $\frac{(x)(x)}{0.5} = 8.0 \times 10^{-16}$

f) $(3x)^3(2x)^2 = 1.08 \times 10^{-3}$

Examples of AP Multiple Choice Questions

Do the problems and answer the questions. You may NOT use a calculator!

1. What is the mole fraction of ethene, C_2H_4, in an aqueous solution that is 28 percent ethene by mass? The molar mass of ethene is 28 g, the molar mass of H_2O is 18 g.

   (a) 0.20
   (b) 0.25
   (c) 0.50
   (d) 0.67
   (e) 0.75

   To solve this problem you will need to solve this:

   $28 \times \frac{1}{28} = x$

   $72 \times \frac{1}{18} = y$

   Then your final answer is found by $\frac{x}{x+y}$

2. If 200. mL of 0.80 *M* $MgCl_2$(aq) is added to 600. mL of distilled water, what is the concentration of Cl^-(aq) in the resulting solution?

   (a) 0.20 *M*
   (b) 0.30 *M*
   (c) 0.40 *M*
   (d) 0.60 *M*
   (e) 1.2 *M*

   To solve this problem you will need to solve this:

   $\frac{(200)(0.80)}{800}$

3. How many grams of calcium carbonate, $CaCO_3$, contain 48 grams of oxygen atoms?

   (a) 41 grams
   (b) 50. grams
   (c) 62 grams
   (d) 88 grams
   (e) 100 grams

   To solve this problem you will need to solve this:

   $48 \times \frac{1}{16} \times \frac{1}{3} \times \frac{100}{1}$

4. When a 1.25-gram sample of limestone, that cotains $CaCO_3$ and inert impurities was dissolved in acid, 0.22 grams of CO_2 was generated. What was the percent of $CaCO_3$ by mass in the limestone?

   (a) 20%
   (b) 40%
   (c) 67%
   (d) 80%
   (e) 100%

   To solve this problem you will need to solve this:

   $0.22 \times \frac{1}{44} \times \frac{1}{1} \times \frac{100}{1} = x \quad then\ final\ answer\ is = \frac{x}{1.25}$

5. A gaseous mixture containing 7.0 moles of hydrogen, 2.5 moles of oxygen, and 0.50 mole of helium at a total pressure of 0.60 atmospheres. What is the partial pressure of the hydrogen?

   (a) 0.13 atm
   (b) 0.42 atm
   (c) 0.63 atm
   (d) 0.90 atm
   (e) 6.3 atm

   To solve this problem you will need to solve this:

   $\left(\frac{7}{10}\right)(0.60)$

6. The density of an unknown gas is 2.00 grams per liter at 3.00 atmospheres pressure and 127 °C. What is the molecular weight of this gas? (R = 0.0821 liter-atm / mole-K)

(a) 254/3 R
(b) 188 R
(c) 800/3 R
(d) 600 R
(e) 800 R

To solve this problem you will need to solve this:

$$\frac{2}{\frac{(3)(1)}{(R)(127+273)}}$$

7. $CH_4(g) + 2\ O_2(g) \rightarrow CO_2(g) + 2\ H_2O(l);\ \Delta H = -890$ kJ

$\Delta H_f°$ $H_2O(l)$ = - 290 kJ / mole

$\Delta H_f°$ $CO_2(g)$ = - 390 kJ / mole

What is the standard heat of formation of methane, $\Delta H_f°$ $CH_4(g)$, as calculated from the data above?

(a) -210. kJ/mole
(b) -110. kJ/mole
(c) -80. kJ/mole
(d) 80. kJ/mole
(e) 210. kJ/mole

To solve this problem you will need to solve this:

$$-890 = [2(-290) + (1)(-390)] - x$$

8. If 70. grams of K_3PO_4 (molar mass 210 grams) is dissolved in enough water to make 250 milliliters of solution, what are the concentrations of the potassium and the sulfate ions?

	$[K^+]$	$[PO_4^{3-}]$
(A)	0.75 M	0.75 M
(B)	1.0 M	2.0 M
(C)	1.3 M	1.3 M
(D)	2.0 M	2.0 M
(E)	4.0 M	1.3 M

To solve this problem you will need to solve this:

$$70 \times \frac{1}{210} = x \qquad [PO_4^{3-}] = \frac{x}{0.25} \qquad [K^+] = \frac{3x}{0.25}$$

9. What mass of Cu is produced when 0.0500 mol of Cu_2S is reduced completely with excess H_2?

(a) 6.35 g
(b) 15.9 g
(c) 24.5 g
(d) 39.4 g
(e) 48.9 g

To solve this problem you will need to solve this:

$$0.0500 \times \frac{2}{1} \times \frac{63.55}{1}$$

10. $CS_2(l) + 3\ O_2(g) \rightarrow CO_2(g) + 2\ SO_2(g)$

What volume of $O_2(g)$ is required to react with excess $CS_2\ (l)$ to produce 2.0 liters of $CO_2(g)$? (Assume all gases are measured at 0°C and 1 atm.)

(a) 6 L
(b) 22.4 L
(c) 1/3 x 22.4 L
(d) 2 x 22.4 L
(e) 3 x 22.4 L

To solve this problem you will need to solve this:

$$2 \times \frac{1}{22.4} \times \frac{3}{1} \times \frac{22.4}{1}$$

11. A 2 L container will hold about 6 g of which of the following gases at 0°C and 1 atm?

(a) SO_2
(b) N_2
(c) CO_2
(d) C_4H_8
(e) NH_3

To solve this problem you will need to solve this:

$$2 \times \frac{1}{22.4} = x \qquad \text{then solve } \frac{6}{x}$$

12.

$$2\ N_2H_4(g) + N_2O_4(g) \rightarrow 3\ N_2(g) + 4\ H_2O(g)$$

When 8 g of N_2H_4 (32 g mol^{-1}) and 46 g of N_2O_4 (92 g mol^{-1}) are mixed together and react according to the equation above, what is the maximum mass of H_2O that can be produced?

(a) 9.0 g
(b) 18 g
(c) 36 g
(d) 72 g
(e) 144 g

To solve this problem you will need to solve these two problems:

$$8 \times \frac{1}{32} \times \frac{4}{2} \times \frac{18}{1} = \_\_\_\_\_ \qquad \text{and} \qquad 46 \times \frac{1}{92} \times \frac{4}{1} \times \frac{18}{1} = \_\_\_\_\_$$

13. What number of moles of O_2 is needed to produce 71 grams of P_4O_{10} from P? (Molecular weight P_4O_{10} = 284)

(a) 0.500 mole
(b) 0.625 mole
(c) 1.25 mole
(d) 2.50 mole
(e) 5.00 mole

To solve this problem you will need to solve this:

$$71 \times \frac{1}{284} \times \frac{10}{1} =$$

Rate = $k[A]^2 [B]$

14. The rate of a certain chemical reaction between substances A and B obeys the rate law above. The reaction is first studied with [A] and [B] each 1 x 10^{-3} molar. If a new experiment is conducted with [A] and [B] each 2 x 10^{-3} molar, the reaction rate will increase by a factor of

(a) 2
(b) 4
(c) 6
(d) 8
(e) 16

To solve this problem you will need to solve this:

$[2]\,^2[2] =$

15. A 0.5-molar solution of a weak monoprotic acid, HA, has a K_a of 3.2 x 10^{-5}. The $[H_3O^+]$ is?

(a) 5.0 x 10^{-7}
(b) 2.0 x 10^{-7}
(c) 1.6 x 10^{-5}
(d) 1.6 x 10^{-4}
(e) 4.0 x 10^{-3}

To solve this problem you will need to solve for x.

$$\frac{(x)(x)}{0.5} = 3.2 \times 10^{-5}$$

16. Uranium-235 undergoes neutron capture as shown in the equation below. Identify nuclide ***X***.

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n}^{0} \longrightarrow ^{145}_{56}\text{Ba} + 3\,^{1}_{0}\text{n}^{0} + X$$

(a) $^{93}_{34}\text{Se}$
(b) $^{90}_{34}\text{Se}$
(c) $^{88}_{36}\text{Kr}$
(d) $^{90}_{36}\text{Kr}$
(e) $^{93}_{36}\text{Kr}$

To solve this problem: the numbers on the top for the reactants must equal the numbers on the top for the products. Same is true for the bottom numbers.

17. $$H_2(g) + Br_2(g) \leftrightarrow 2\ HBr(g)$$

At a certain temperature, the value of the equilibrium constant, K, for the reaction represented above is 4.0×10^5. What is the value of K for the <u>reverse</u> reaction at the same temperature?

(a) 4.0×10^{-5}
(b) 2.5×10^{-6}
(c) 2.5×10^{-5}
(d) 5.0×10^{-5}
(e) -4.0×10^{5}

To solve this problem you will need to solve

$$\frac{1}{4.0 \times 10^5}$$

18. If the acid dissociation constant, K_a, for an acid HA is 8×10^{-4} at 25 °C, what percent of the acid is dissociated in a 0.50-molar solution of HA at 25 °C?

(a) 0.08%
(b) 0.2%
(c) 1%
(d) 2%
(e) 4%

To solve this problem you will need to solve for x in the first equation and then plug it into the second expression and solve.

$$\frac{(x)(x)}{0.5} = 8 \times 10^{-4}$$

$$\frac{x}{0.5} \times 100$$